Isotropic Positive Definite Functions on Spheres Generated from Those in Euclidean Spaces

Zhihui Nie [1]

[1]Eastern Nazarene College, Quincy, MA

There has been renewed interest in isotropic positive definite functions in the Euclidean space R^d or on the sphere S^d, with applications in approximation theory, machine learning, probability, and spatial statistics. This presentation will give an idea of how to construct an isotropic positive definite function on spheres from Euclidean space. For a continuous function $g(x)$ on $[0, \pi]$ with $g(\pi) = 0$, if it satisfies the inequality

$$\int_0^\pi u^{\alpha+\frac{1}{2}} g(u) J_{\alpha-\frac{1}{2}}(xu) du \geq 0, \qquad x \geq 0,$$

it is shown in this presentation that

$$\int_0^\pi g(u) P_n^{(\alpha)}(\cos u) \sin^{2\alpha} u du \geq 0, \qquad n \in N_0,$$

where α is a non-negative integer, and $J_\nu(x)$ and $P_n^{(\nu)}(x)$ denote the Bessel function and the ultraspherical polynomial, respectively. As a consequence, an isotropic and continuous positive definite function in the Euclidean space, if it is compactly supported, it can be adopted as an isotropic positive definite function on a sphere.